Solve a quartic equation

Solve $(x-3)^4 + (x-5)^4 = 2018$ for real and complex roots.

Note that the equation is power 4, not power 2.
Any easy way to solve?

Put $u = x - 4$, the equation becomes

$$(u+1)^4 + (u-1)^4 = 2018$$

$$2u^4 + 12u^2 + 2 = 2018$$

$$u^4 + 6u^2 + 1 = 1009$$

$$u^4 + 6u^2 - 1008 = 0$$

$$(u^2)^2 + 6(u^2) - 1008 = 0$$

By quadratic equation formula,

$$u^2 = -3 \pm 3\sqrt{113}$$

$$u = \pm\sqrt{-3 \pm 3\sqrt{113}}$$

$$x - 4 = \pm\sqrt{-3 \pm 3\sqrt{113}}$$

$$\therefore x = 4 \pm \sqrt{-3 + 3\sqrt{113}},\ 4 \pm i\sqrt{3 + 3\sqrt{113}}$$

Figure out why we can use the $u = x - 4$.

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